# Information costs in financial markets: evidence from the Tunisian stock market

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## Abstract

**Purpose** – The purpose of this paper is to discuss a widespread idea in the financial literature: information in financial markets is free. Indeed, whenever an investor wants to intervene to purchase and/or to sell, he/she faces the need to access the information, which he/she judges to ensure an optimal decision.

**Design/methodology/approach** – The paper uses the entropy statistics in order to estimate the information cost of the assets of the Tunisian stock market over the period extending from 2002 to 2005.

**Findings** – The obtained results show that the information costs follow a Brownian motion. This finding lends empirical support to the theoretical position that has always been adopted in the relevant literature: in finance, as in economy, the majority of the series follow a Brownian motion.

**Practical implications** – The proposed methodology offers investors the opportunity to estimate the information cost by taking into account the quotation probability, a simple approach that can be used not only by fund managers, but also by financial market investors.

**Originality/value** – The paper uses entropy as a relatively new tool applied in financial theory. It offers a new understanding of information cost. The paper will be of interest for financial market investors and academics.

**Keywords** Information management, Cost accounting, Tunisia, Stock markets

**Paper type** Research paper

## 1. Introduction

The asset pricing model is as a puzzle, where more and more of parts get involved in the definition of the general balance on the financial markets. This analogy (with a puzzle) is inspired by the definition proposed by Black (1976), which specifies the choice of dividend policy and Myers (1984), which identifies the choice of capital structure. In this study, we propose to analyze one of these "parts": the information cost.

Nowadays, the flow of new information which reaches the investors is substantial, especially with the development of internet information services. Barber and Odean (2001) state that:

> The Internet changed the way in which information is subjected to investors and the ways with which these investors react to this information. The Internet made it possible to reduce, as well the fixed as the marginal costs of production of the financial services.

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Peress (2005) remarks that:

> In the US, the cost of trading individual stocks (including commissions and bid-ask spreads) has dropped dramatically since commissions were deregulated in 1975 (Jones, 2002). Mutual fund fees (including fund expenses, loads, and distribution costs) have also decreased steadily from 2.26% in 1980 to 1.35% in 1998 (Rea *et al.*, 1999). The picture is less clear for information costs. On one hand, the media have increased their coverage of the stock market since the 1970s and, more recently, the Internet has given investors instant access to information on companies. On the other hand, finding relevant, quality information in this ocean of facts and commentary is a daunting task.

The information cost is not only restricted to the purchasing of information, but also and more specifically it covers the cost of analysis, treatment, and classification of information, a cost which is not to be neglected. In addition, authors such as Mohammed and Yadav (2002), Peress (2005), and Lundtafte (2006) emphasize the fact that the available information is not of the same quality. Investors are always in search of high-quality information, so they agree to pay additional costs.

At this stage, a major problem arises: how it is possible to estimate the information cost? To address this problem, the present paper proposes to use the entropy statistics with an application to the Tunisian stock market's (TSM) assets over the period extending between 2002 and 2005. Therefore, we try to identify the nature of the series of the information cost, and more particularly, to seek to determine if it follows a Brownian motion.

The organization of this paper is as follows: Section 2 is concerned with the description of the entropy statistics and its applications in the financial field. Section 3 proposes an estimation of the information cost of the assets of the TSM over the period extending between 2002 and 2005. In Section 4, we try to check if the information cost follows a Brownian motion.

## 2. Entropy statistics

The concept of entropy was first introduced by physicians in order to describe complex systems: heat engines initially, then gases and transitions across phases. The entropy is regarded as a measure of the disorder which can be inherent in the system (gas) or which can result from a description of the system (disturbed reception of a complex signal).

Szilard (1929) was the first to be interested in the physics-information relation. More recently, authors such as Peng (2005), Chen (2004), Van Nieuwerburgh and Veldkamp (2004), Dadpay *et al.* (2007), Clarke (2007), Gajdos *et al.* (2008), Sbuelz and Trojani (2008), and Zapart (2009) have introduced the entropy statistics in relation to the information theory.

Generally, the entropy statistics is formulated as follows: "In the information theory, uncertainty on a random variable $X$, whose continuous density function of probability $f(x)$ can be measured by its entropy as follows:

$$H(X) = -E\log[f(X)] = -\int(\log f(x))f(x)dx \tag{2.1}$$

If $X$ follows a normal distribution $(N(\overline{X}, \sigma^2))$, its entropy depends only on the logarithm of the parameter of volatility:

$$H(X) = \log\sigma + 0.5\log(2\pi e) \tag{2.2}$$

where $e$ represents the base of the exponential function.

The conditional entropy of $X$ compared to another random variable $Y$ is the anticipated entropy of the conditional distribution. If $f(x, y)$ is the joined function of density of probability of $X$ and $Y$, $f(X|Y)$ is the conditional density of $X$ knowing $Y$ and $g(y)$ is the function of marginal density of $Y$, thus, the conditional entropy of $X$ knowing $Y$ is:

$$H(X|Y) = \int H(X|y)g(y)dy = -\iint \log f(x|y)f(x,y)dxdy \quad (2.3)$$

The proportion (or quantity) of information that $Y$ contains on $X$ is defined as being the reduction of the uncertainty of $X$ due to the knowledge of $Y$:

$$I(X;Y) = H(X) - H(X|Y) = \iint f(x,y)\log\frac{f(x,y)}{f(x)g(y)}dxdy \quad (2.4)$$

An interesting property of this measurement of information is that it remains unchanged in spite of the change of any non-null linear transformations on the two variables $X$ and $Y$."

Chen (2004), for example, notes that the entropy statistics is meant to quantify the value of information (quantity which makes it possible to make a decision) by a random variable. In other words, according to the entropy statistics, the value of information is a function of the probability. It satisfies the whole of these properties:

- The information value of two events is higher than the value of each of them.
- If two events are independent, the information value of the two events will be the sum of the two.
- The information value of any event is non-negative.

Thus, the only mathematical functions that satisfy all the above of these properties are of the form:

$$H(P) = -\log_b P \quad (2.5)$$

with:

$P$: is the probability associated with a given event.

$b$: is a positive constant. Generally, it is equal to two.

This measurement is used in several fields of economy and finance. It is possible to reexamine the purpose of Maasoumi and Racine (2002), for example.

The following section proposes an application of the entropy statistics, which is carried out to estimate the information cost of assets quoted on the TSM.

## 3. Estimation of information cost

The information cost is estimated by using the entropy statistics as defined by equation (2.5).

The sample selected for the purposes of the present study includes all of the assets quoted on the TSM and which were present over the period extending from 2002 to 2005. However, not all of the included assets were present over all this period. For instance, BATAM disappeared since the 24 January 2003, while some other assets, such as ASSAD, GIF, and KARTHAGO, emerged during this period. Assets like CARTE, which were rarely present on the market during this period, were eliminated from the sample. So the final sample consists of a total number of 35 assets and 25,375 observations.

For a given asset, we compute the information cost, which the investor has to spend at the beginning of the month.

Concretely, for a given asset, the quotation probability relating to one month is calculated. For this purpose, we use the binomial distribution given by:

$$P[X = x] = C_n^x p^x (1 - p)^{n-x} \tag{3.1}$$

with:

$n$ : the total number of the working days on the TSM for a given month.

$x$ : the number of days when the asset is present for the same month.

$p$ : the success probability of the asset for the same month.

Table I contains results of all assets. For example, for the asset *AB* on January 2002, we note: $n = 22$, $x = 19$, and $p = 19/22 = 0.863$. The quotation probability is $P[X = 19] = 0.240$ and the information cost is $H(0.2409) = 2.053$. The same reasoning is applied for the remainder of the period. Then the average information cost for the asset *AB* corresponding to 2002-2005 is 2.300.

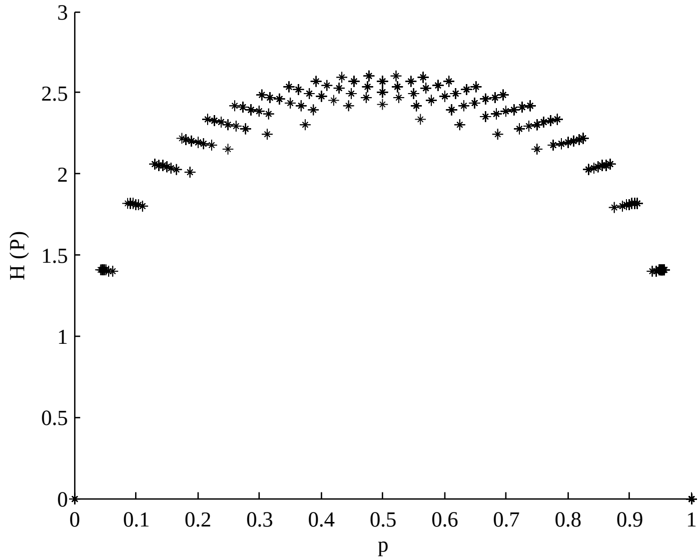
In order to clarify the relation between the success probability ($p$) and the information cost ($H(P)$), the following figure is provided (Figure 1).

From this figure, it is possible to notice that the relation between the information cost and the success probability increases steadily until it reaches $p = 0.5$, at which point it begins to decrease.

Considering the facts from a financial point of view, it is possible to advance the following explanation: when an asset is not long present on the market ($p$ tends to 0), investors are not interested in buying and/or selling it, hence the information cost tends towards 0. In that case, where the success probability increases (but remains lower than 0.5), investors are likely to request information to optimize their investment decisions, consequently, the information cost increases. On the other hand, if the success probability increases, and goes higher than 0.5, this suggests that the asset is

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Asset | AB | ALKIMIAA | ATB | ATL | BH | BIAT | BNA |
| Information cost | 2.300 | 2.169 | 2.244 | 2.032 | 1.669 | 1.384 | 2.335 |
| Asset | BS | BT | BTB | $\alpha$L | ELECTRO.S | GLS | I$\alpha$F |
| Information cost | 1.730 | 1.076 | 1.356 | 2.354 | 1.001 | 2.035 | 2.107 |
| Asset | MSZRAA | MG | MONOPRIX | SFBT | SIAME | SIMPAR | SIPHAT |
| Information cost | 2.264 | 1.670 | 2.198 | 0.134 | 1.835 | 1.958 | 0.382 |
| Asset | SOTETEL | SOTRAPIL | SOTUMAG | SOTUVER | SPDIT | STAR | STB |
| Information cost | 0.347 | 0.462 | 2.207 | 2.230 | 1.754 | 2.054 | 1.033 |
| Asset | STEQ | TUN LAIT | TUN LEASING | TUN INVEST | TUNISAIR | Ub$\alpha$ | UIB |
| Information cost | 0.661 | 1.856 | 2.033 | 2.403 | 0.558 | 2.327 | 1.105 |

**Table I.**
Average information cost (2002-2005)




**Notes:** 1,680 points are shown in this figure; they represent observations for the 35 assets observed in the all 48 months

**Figure 1.** Relation between the success probability and the information cost

quite present on the market. In this case, and under the effect of the supply and demand law, the asset price is itself a source of free information. This last conclusion can be theoretically accounted for by reference to Grossman and Stiglitz's (1980) model, which proposes that the equilibrium price, assumed to be fully revealing, can by itself reveal all the information that is available on the market. Thus, an investor is not compelled to expend money to purchase information, but he can extract freely the information that he needs by reference to the equilibrium price of the asset.

In this context, this extreme case is only possible if $p = 1$, which means that the asset is always present on the market, a case which we have only once in our analysis (SFBT on 2005). In addition, as long as $0.5 \leq p < 1$, an investor has to buy information, but the information cost decreases when $p$ tends towards 1 (a case which obtains when the price is fully revealing). In the opposite case (where $p$ moves away from one but remains higher than 0.5), the price is expected to be partly revealing. In this case, the price's informative content is not significant enough (Muendler, 2003) and some investors would prefer to purchase information in order to make better investment decisions.

In summary, it is possible to advance the four following situations:

(1) If the success probability of an asset is equal to 0, this means that the asset is completely absent from the market, generated by the fact that investors do not express any interest in this asset. Consequently, no investor is ready to spend money to acquire information and, therefore the information cost remains equal to 0.

(2) If the success probability of an asset is equal to 1, this means that the asset is always present on the market and that its price is fully revealing, under the effect of the supply and demand law. In this case, any investor can have access

to free information contained in the equilibrium price and, therefore, is not obliged to expend money to have information on this asset; consequently, the information cost is equal to 0.

(3) If the success probability of an asset varies between 0 and 0.5, the asset is not present enough on the market and its equilibrium price is partly revealing. In this case, an investor has "to spend" to have information on this asset. Hence, the information cost increases and it reaches its peak at the point $p = 0.5$.

(4) If the success probability of an asset ranges between 0.5 and 1, the asset is rather present on the market and its equilibrium price tends to become fully revealing. In this case, the information cost decreases and it reaches its minimum level at the point $p = 1$.

In the next section, we try to check if these information costs follow a Brownian motion.

## 4. Information cost and Brownian motion

The Brownian motion is a stochastic process which appears in quasi all the models of all fields dealing with noisy signals, approximation of the networks of the strongly charged queues. In particular, the financial theory largely describes this motion in full detail. Thus, assets, indices, and exchange rates, for example, are often modeled by the geometric Brownian motion.

Hurst (1951) introduced the "Hurst parameter" (noted, in follows by *Hu*) in order to classify the time series according to their structure of dependence. Generally *Hu* is between 0 and 1. If $0.5 < Hu < 1$, we can consider that the series present long memory. The closer $H$ is to 1, the stronger the long memory. If $0 < Hu < 0.5$, we can consider that the series is anti-persistent. If $Hu = 0.5$, the series does not present any long memory and it is simply the geometric Brownian motion.

In order to estimate the Hurst parameter, Mignon (1998), for example, proposes that it is possible to consider the statistics $R/S$, expressed as:

$$Q_T = R/S_T = \frac{1}{\left[\frac{1}{T}\sum_{j=1}^{T}\left(\Delta\hat{z}_j - \overline{\Delta\hat{z}}\right)^2\right]^{1/2}} \times \left[\underset{1\le k\le T}{\mathrm{Max}}\sum_{j=1}^{k}\left(\Delta\hat{z}_j - \overline{\Delta\hat{z}}\right) - \underset{1\le k\le T}{\mathrm{Min}}\sum_{j=1}^{k}\left(\Delta\hat{z}_j - \overline{\Delta\hat{z}}\right)\right] \tag{4.1}$$

where:

$\Delta\hat{z}_t$ : A time serie, $t = 1, \ldots, T$.

$\overline{\Delta\hat{z}}$ : The arithmetic average of $\Delta\hat{z}_t$.

This statistics is proportional to $T^H$, where $H$ is the Hurst parameter defined as:

$$H \cong \frac{\log Q_T}{\log T} \tag{4.2}$$

By analogy, we can estimate *Hu* by:

$$Hu = \left|\frac{\log H(P)}{\log P}\right| \tag{4.3}$$

$H(P)$ (information cost) and $P$ (quotation probability) are computed as in the previous section.

Table II and Figure 2 summarize the results which are explained below.

It is possible to note that the majority of the values of *Hu* are equal to $0.5 \pm \epsilon$, which makes it possible to confirm the existence of a geometric Brownian motion. In other words, the information cost follows a geometric Brownian motion similar to several other financial series, the most famous of which are the applications of Black and Scholes (1973) for the valorization of the options prices.

Thus, the information cost can be formalized as follows:

$$X_t = X_0 \exp\left\{\left(b - \frac{1}{2}\sigma^2\right)t + \sigma B_t\right\} \tag{4.4}$$

with: $b$ and $\sigma$: two constants; $B$: standard Brownian motion.

However, it should be noted that by reference to Table I, it can be noticed that there are some values of Hu (for the SIPHAT and SOTRAPIL assets), which are at the level

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Asset | AB | ALKIMIAA | ATB | ATL | BH | BIAT | BNA |
| Hu | 0.513 | 0.507 | 0.519 | 0.488 | 0.471 | 0.451 | 0.521 |
| Asset | BS | BT | BTB | $\alpha$L | ELECTRO.S | GLS | I$\alpha$F |
| Hu | 0.473 | 0.447 | 0.458 | 0.521 | 0.420 | 0.500 | 0.504 |
| Asset | MSZRAA | MG | MONOPRIX | SFBT | SIAME | SIMPAR | SIPHAT |
| Hu | 0.514 | 0.487 | 0.505 | 0.412 | 0.485 | 0.498 | 0.384 |
| Asset | SOTETEL | SOTRAPIL | SOTUMAG | SOTUVER | SPDIT | STAR | STB |
| Hu | 0.411 | 0.369 | 0.505 | 0.506 | 0.485 | 0.497 | 0.459 |
| Asset | STEQ | TUN LAIT | TUN LEASING | TUN INVEST | TUNISAIR | UB$\alpha$ | UIB |
| Hu | 0.434 | 0.503 | 0.483 | 0.522 | 0.432 | 0.514 | 0.437 |

**Table II.**
Average Hurst parameter (2002-2005)

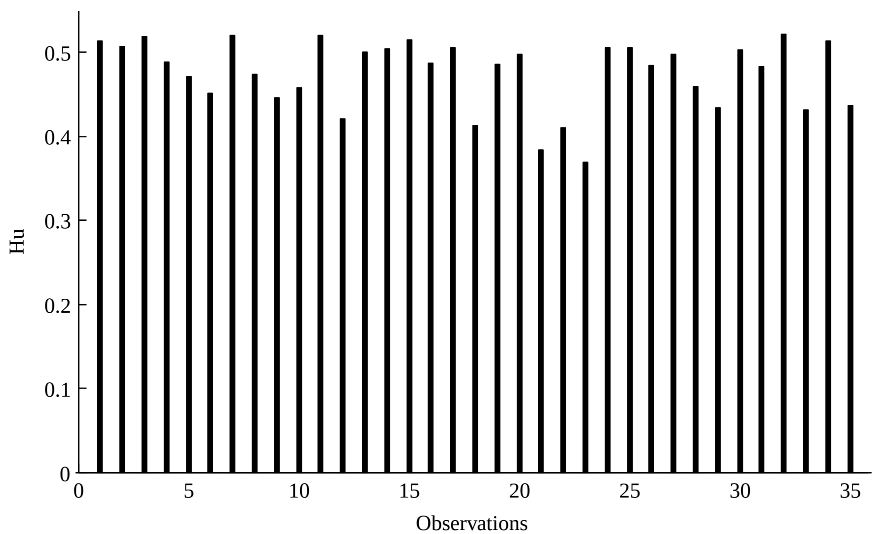



**Figure 2.**
Hurst parameter

of 0.35-0.37. In fact, the success probability of these assets is, respectively, equal to 0.984 and 0.982, a probability which is very close to 1. In these cases, and as specified in the previous section, the information cost tends towards 0. Under such conditions, we cannot consider a geometric Brownian motion for series which take 0 values.

## 5. Conclusion

The goal of our paper is to analyze the role that the information cost plays on the financial markets. Information is not always free, as it has long been assumed. Indeed, the current paper has shown that there are cases where the investor faces an urgent need to expand money on indispensable information in order to make an optimal investment decision. Such cases suggest that prices are partly revealing, which means that equilibrium prices cannot, alone, reflect all the potential information about the assets; hence the need for further information and the existence of information costs. The present paper has proposed an empirical method to the study of information cost as it has demonstrated that these costs can be estimated by the entropy statistics. It has exemplified the usefulness of this method by applying it to the case of financial markets, namely the TSM. The second contribution of this paper lies in determining the precision of the nature of the process of the series of information cost, which is a geometric Brownian motion. These findings lend empirical support to the theoretical position that has always been adopted in the relevant literature: in finance, as in economy, the majority of the series follow a Brownian motion, since Bachelier (1900) until Black and Scholes (1973), and that continues.

It will be interesting to test the impact of the information cost on the asset's price. Several financial models such as those proposed by Grossman and Stiglitz (1980) and Biais (1993) suppose free information. However, the information cost exists and it cannot be neglected. Then an asset-pricing model under costly information is likely to better explain various phenomena observed in financial markets.

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